



# A Framework for Analysing Mechanical-Electronic Composite Power Systems

**Dr Yunjie Gu**

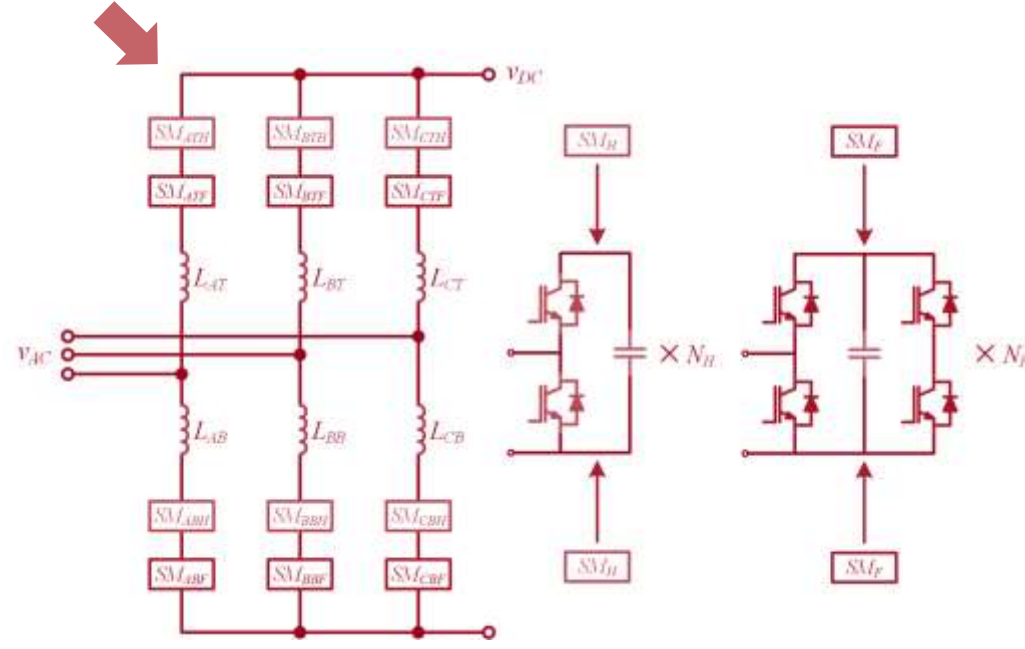
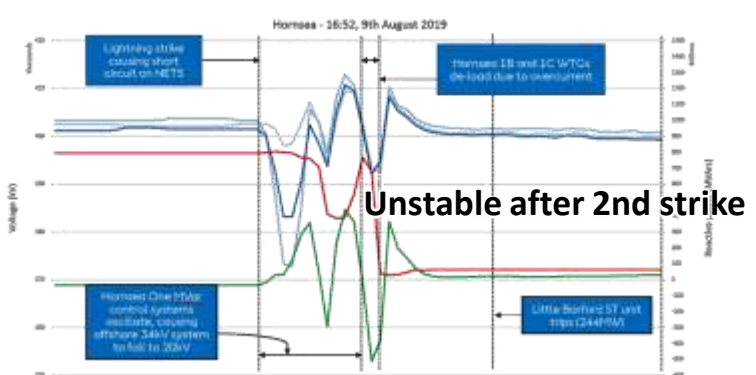
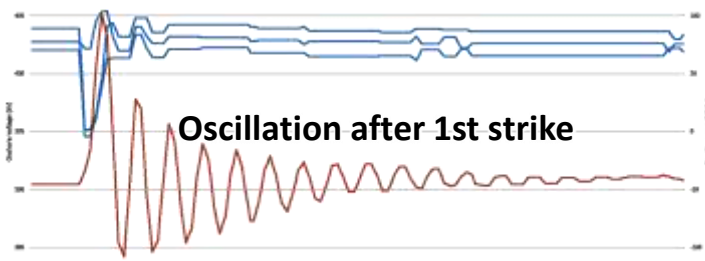
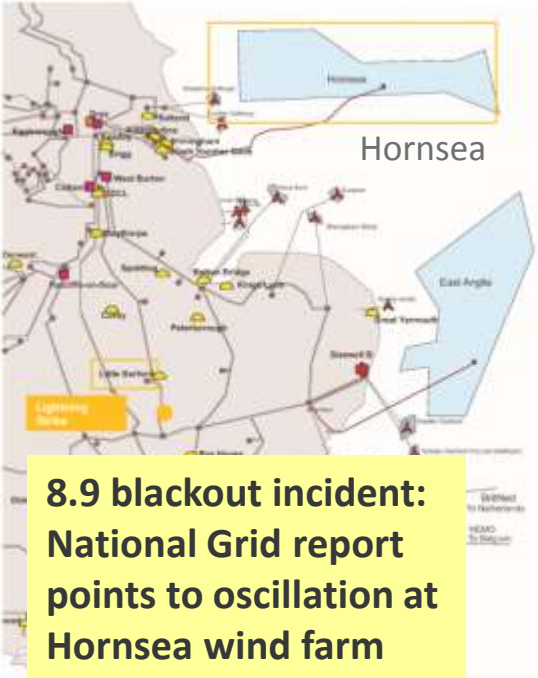
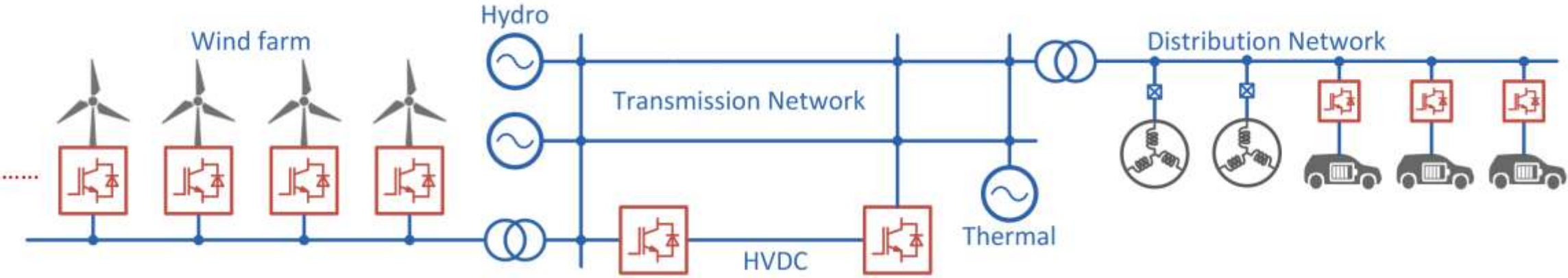
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# More-Electronic Power Systems: A Pandora's Box?



A single converter with many (>100) sub-modules

# Power Electronic Timeline



1930s

Mercury Arc Valve  
HVDC



1960s

Diode  
Rectifier



1970s

Thyristor  
Exciter & HVDC



1980s

MOSFET  
Power Supply

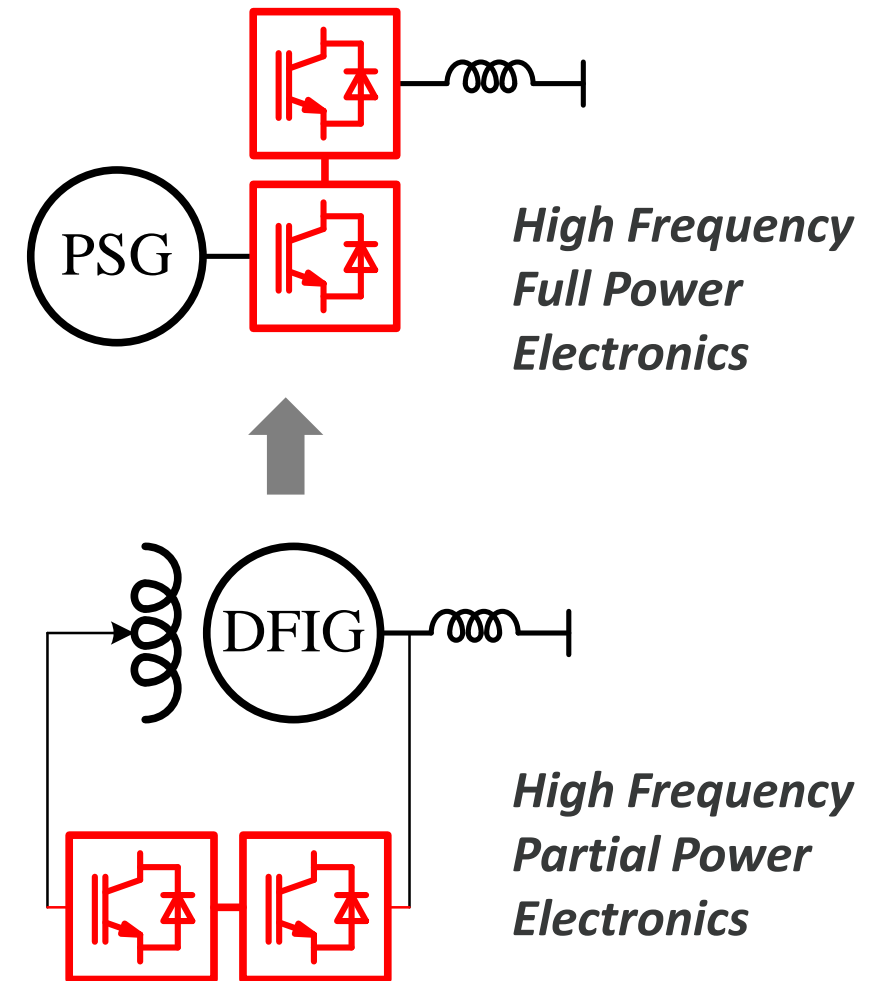
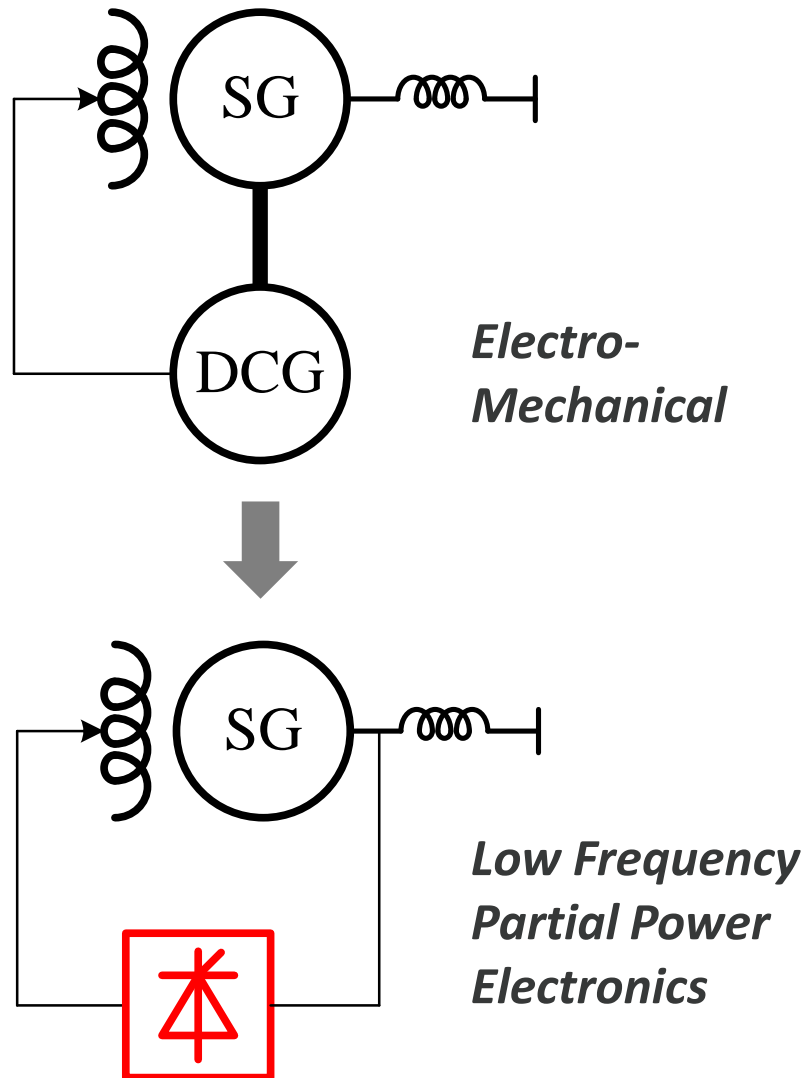


2000s

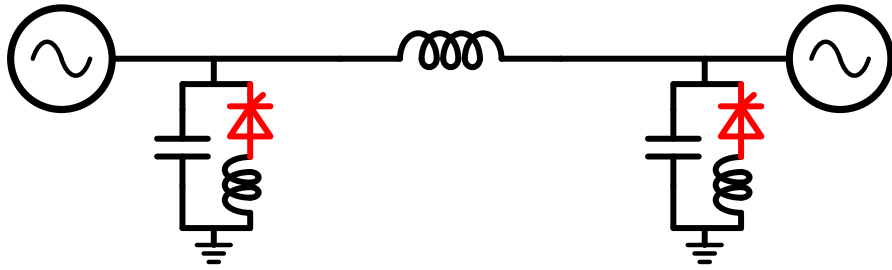
IGBT  
Inverter & HVDC

- *Thyristor Exciters are widely used in power generation since **1970s***
- *MOSFET Power Supplies are widely used in power consumption since **1980s***
- *Why we start talking about more-power-electronic grid now, not thirty years ago?*

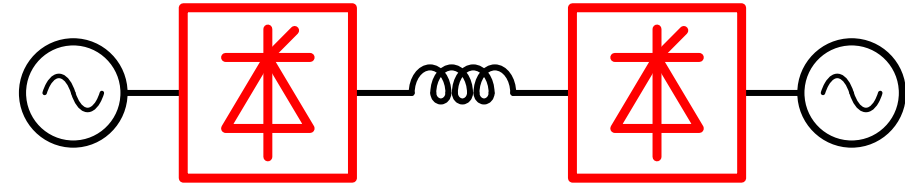
# The Rise of Electronics in Power Generation



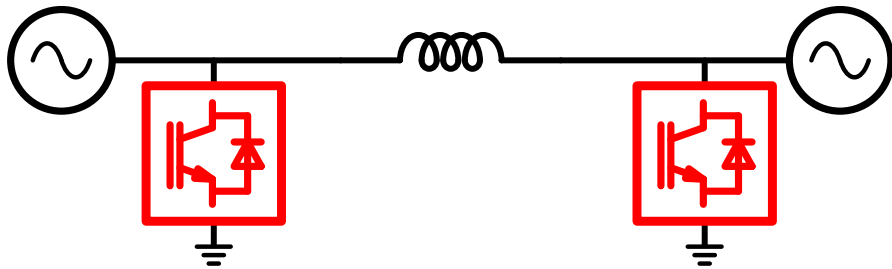
# The Rise of Electronics in Power Transmission



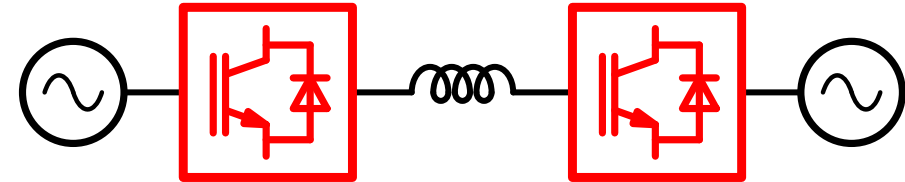
*Low-Frequency Partial-Power Electronics*



*Low-Frequency Full-Power Electronics*



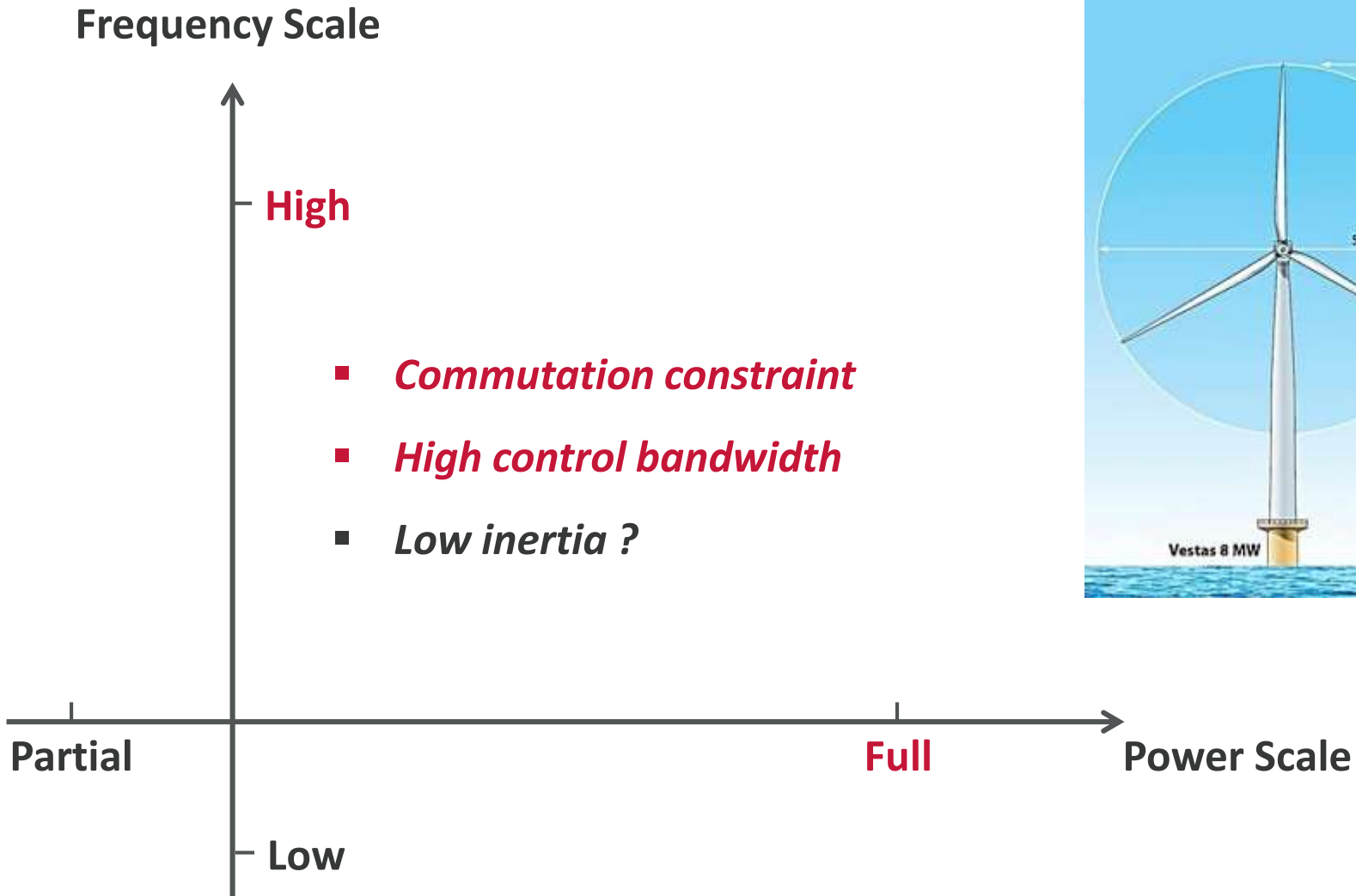
*High-Frequency Partial-Power Electronics*



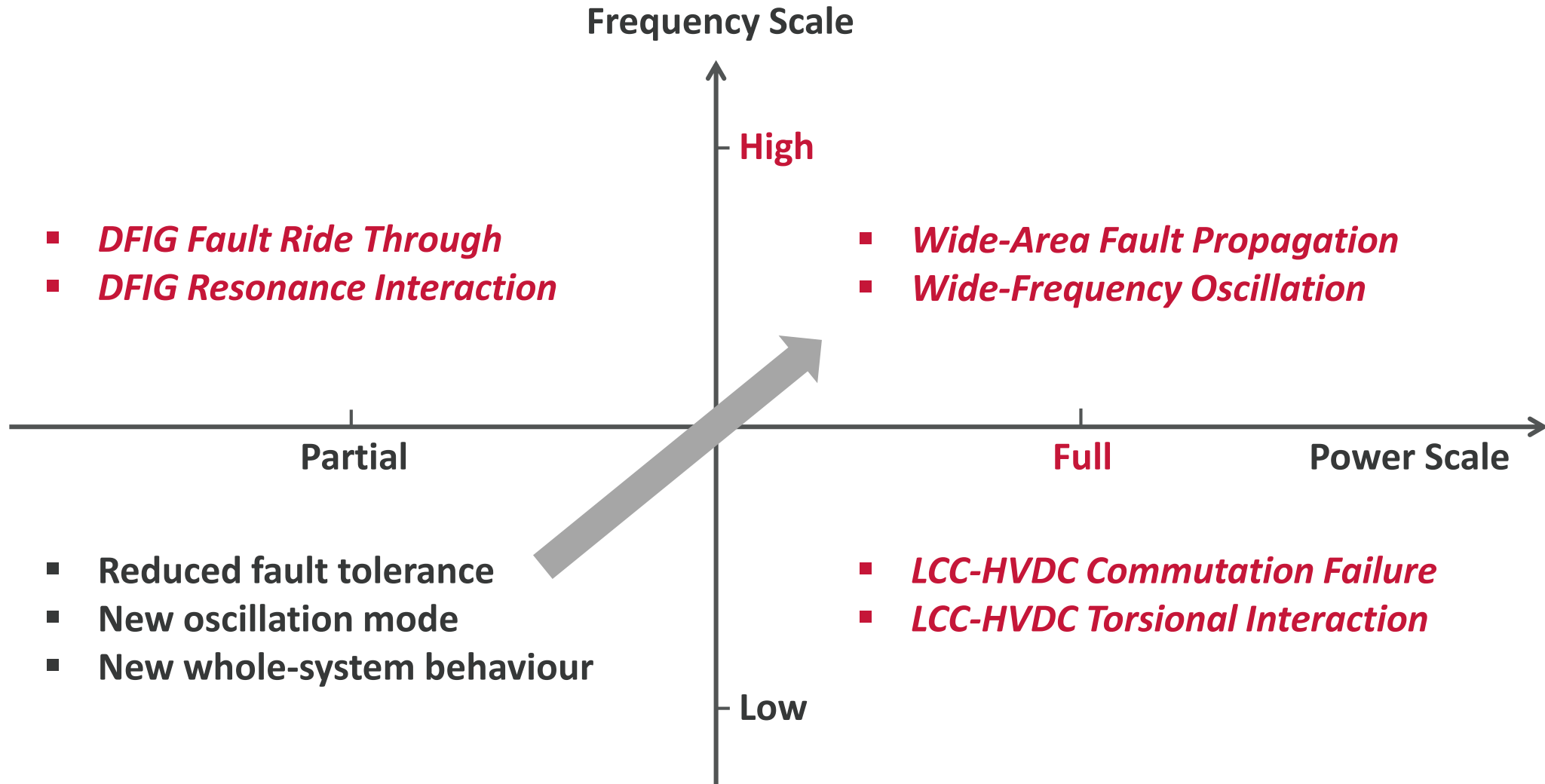
*High-Frequency Full-Power Electronics*



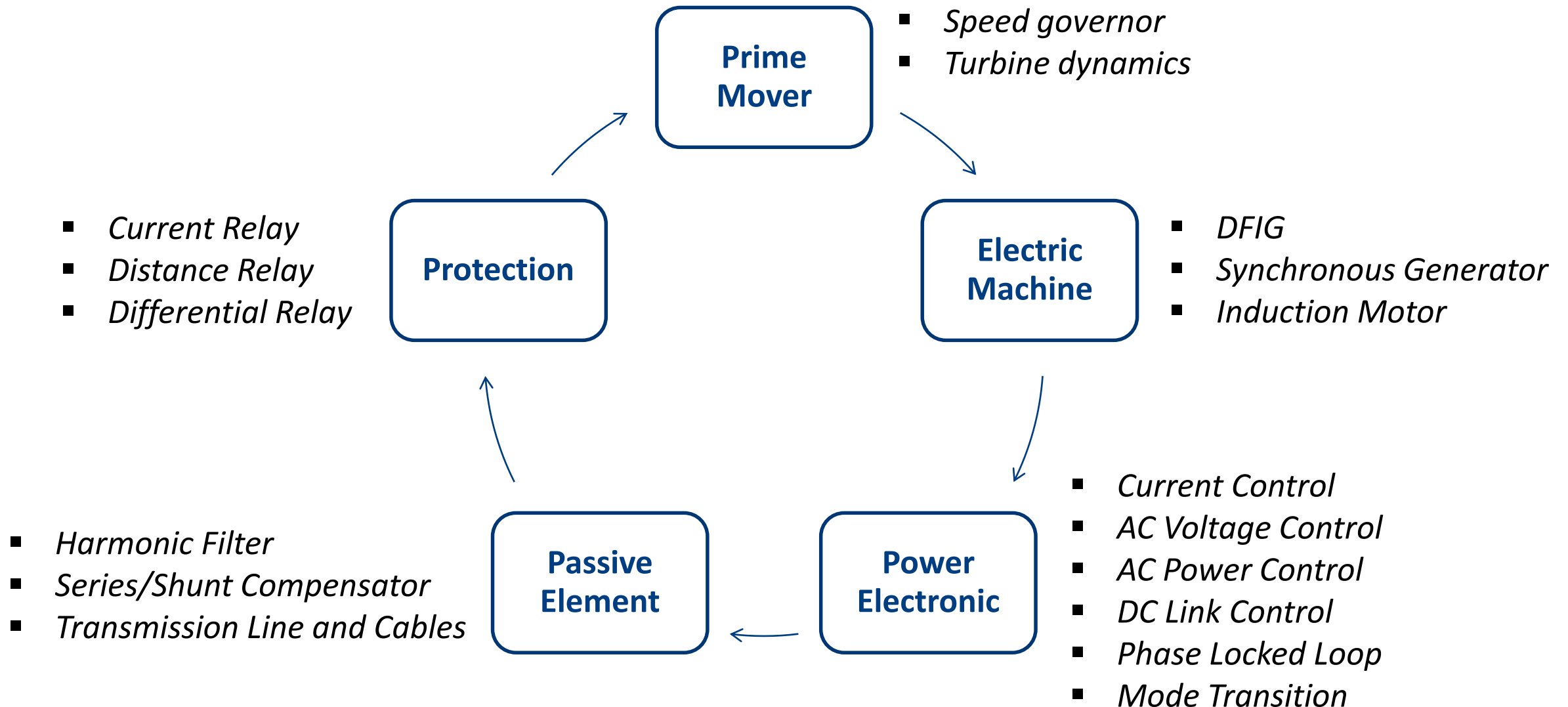
# What Makes the Difference?



# What Are the Consequences?

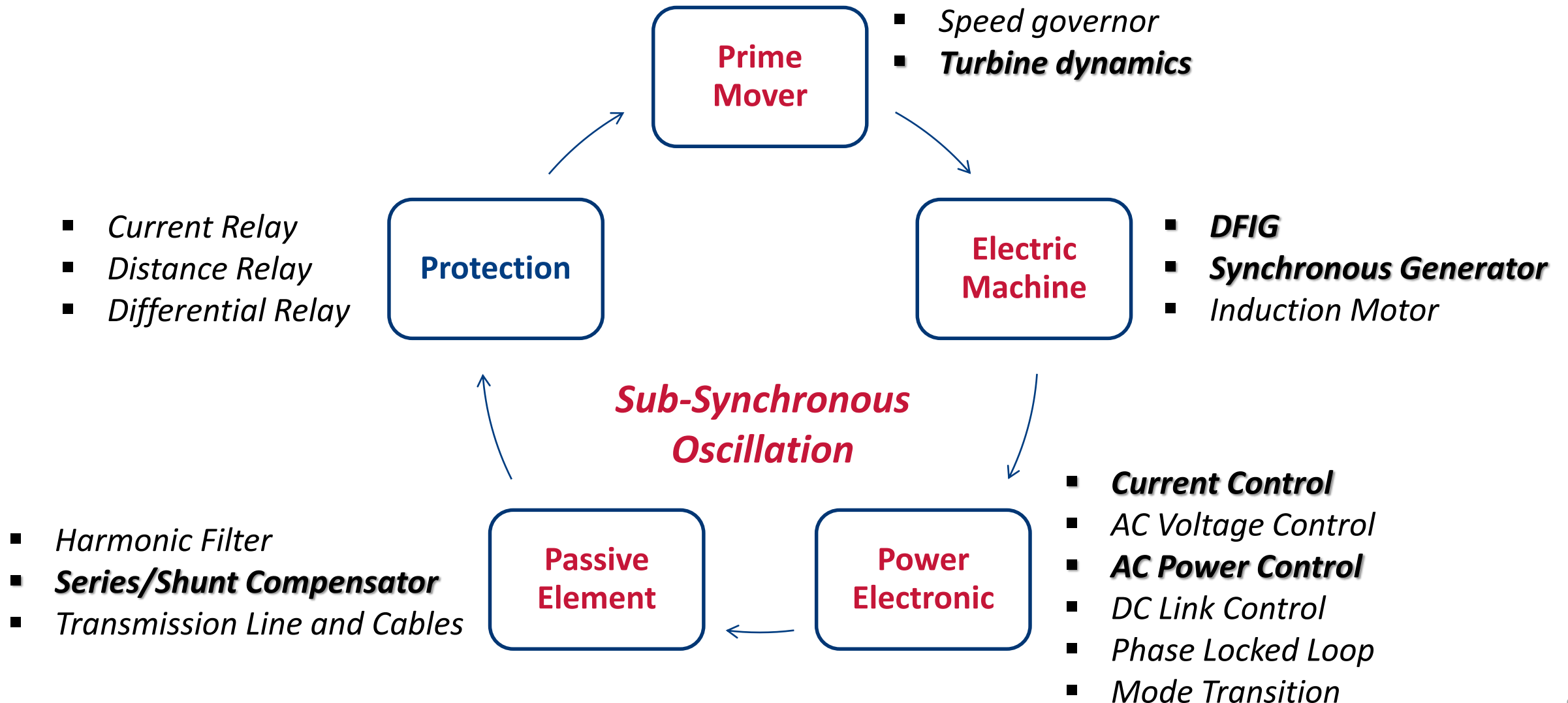


# Whole System Interaction

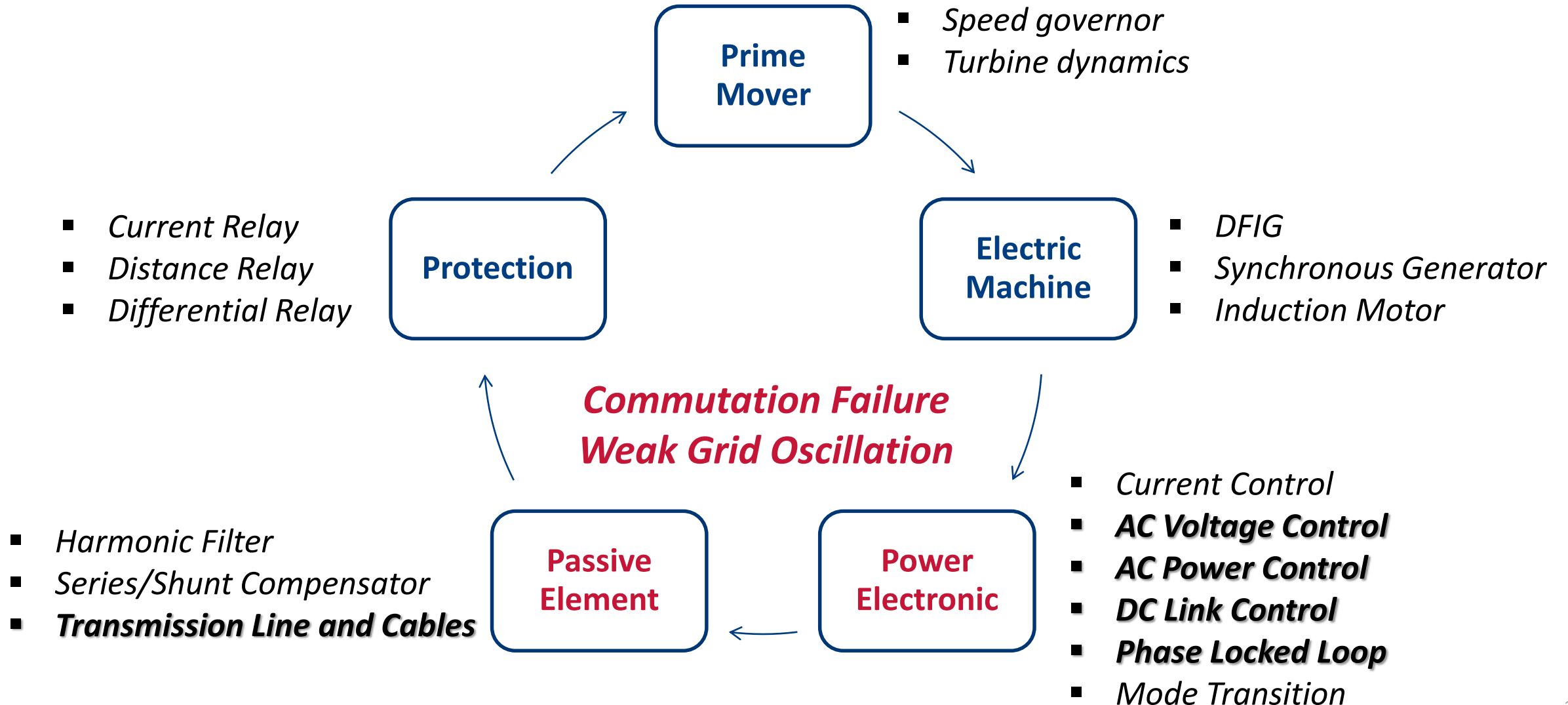




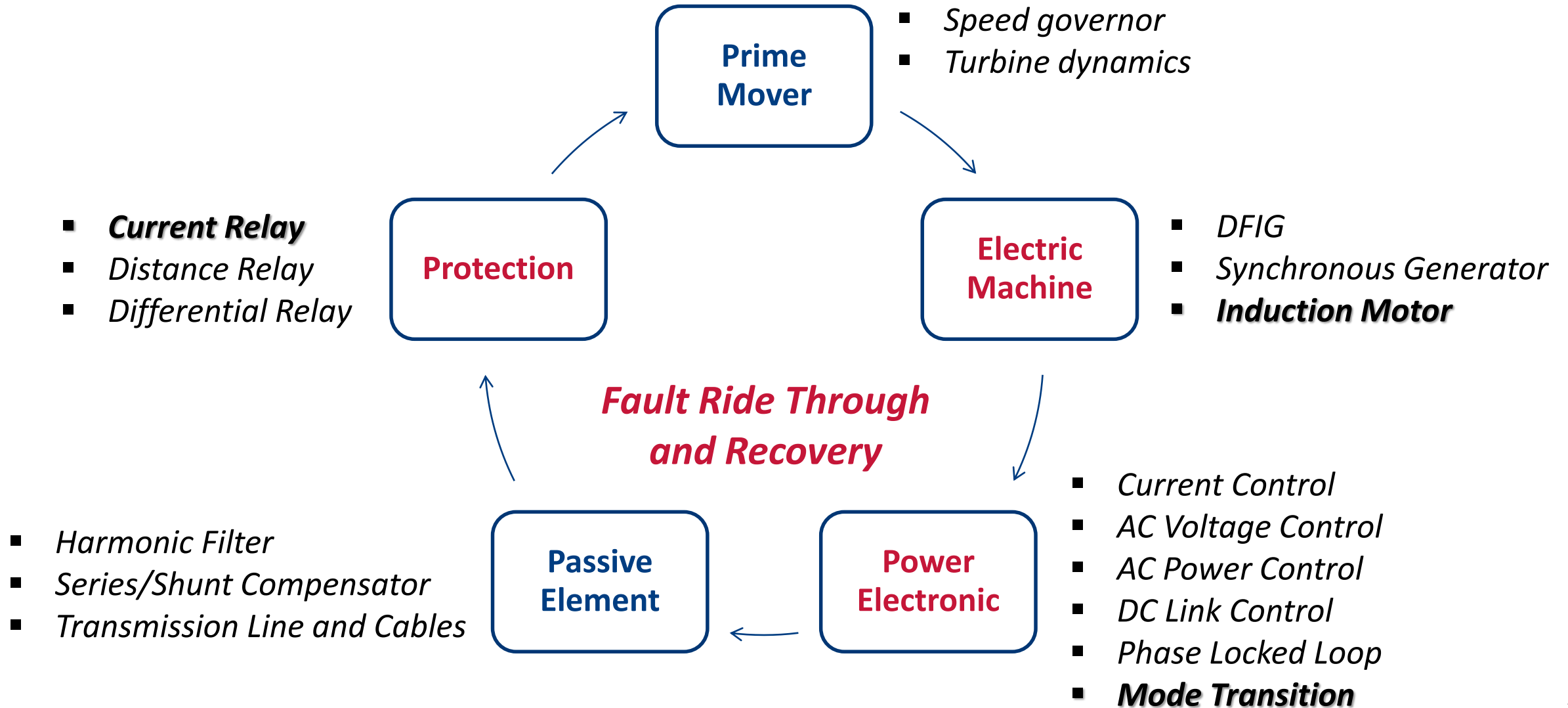
# Whole System Interaction



# Whole System Interaction



# Whole System Interaction



# Whole System Interaction

- **Current Relay**
- *Distance Relay*
- *Differential Relay*

**Protection**

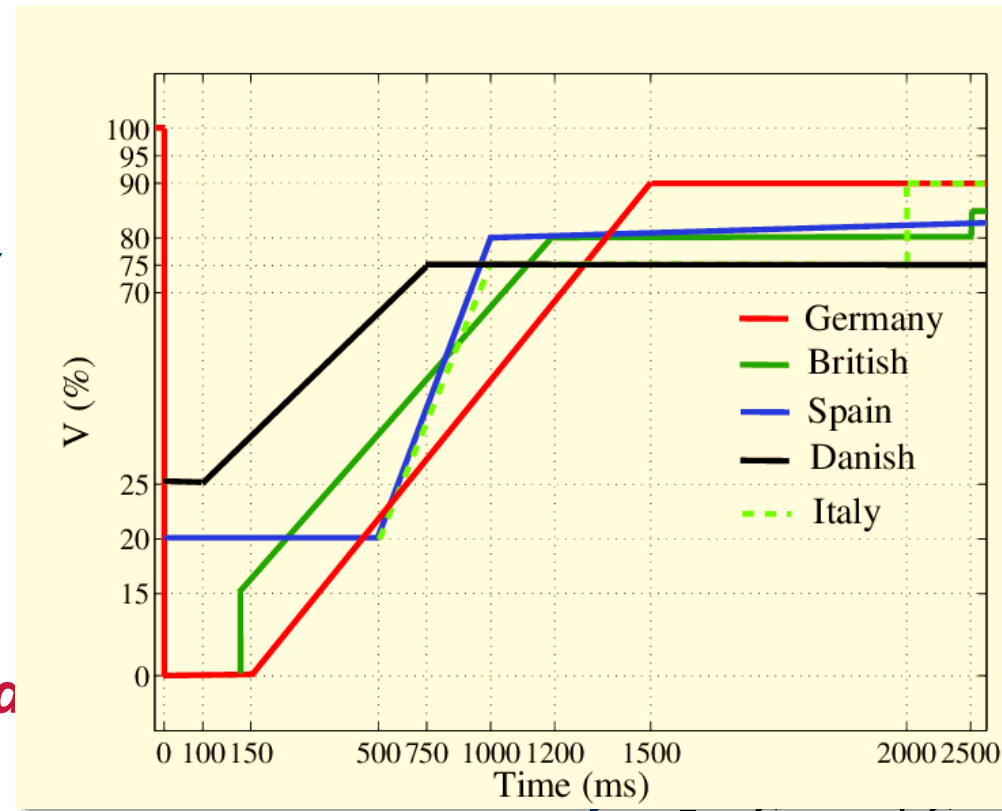
**Fault**

- *Harmonic Filter*
- *Series/Shunt Compensator*
- *Transmission Line and Cables*

**Passive  
Element**

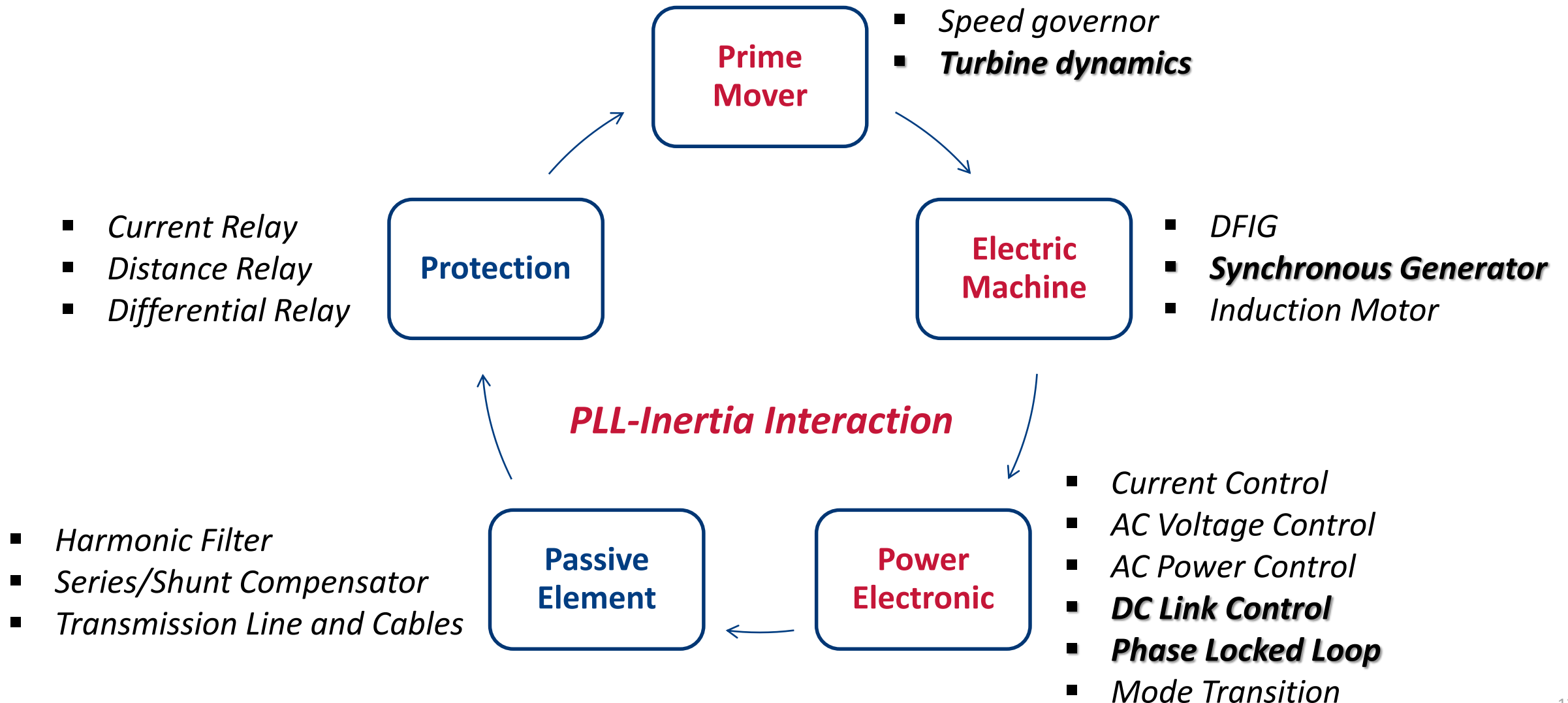
**Power  
Electronic**

- *Current Control*
- *AC Voltage Control*
- *AC Power Control*
- *DC Link Control*
- *Phase Locked Loop*
- **Mode Transition**



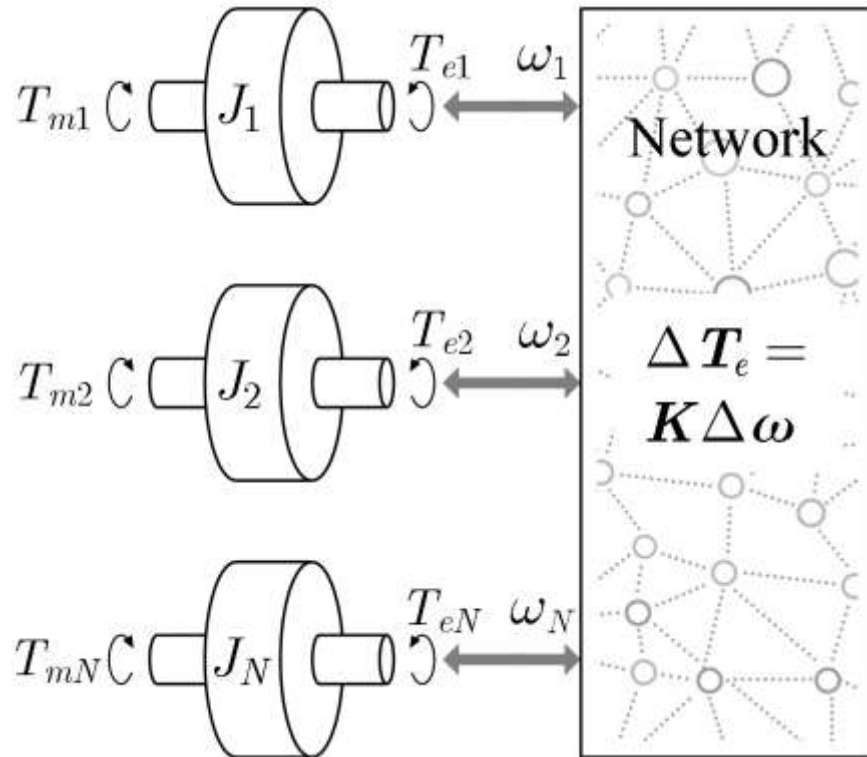
**Synchronous Generator  
Protection Motor**

# Whole System Interaction

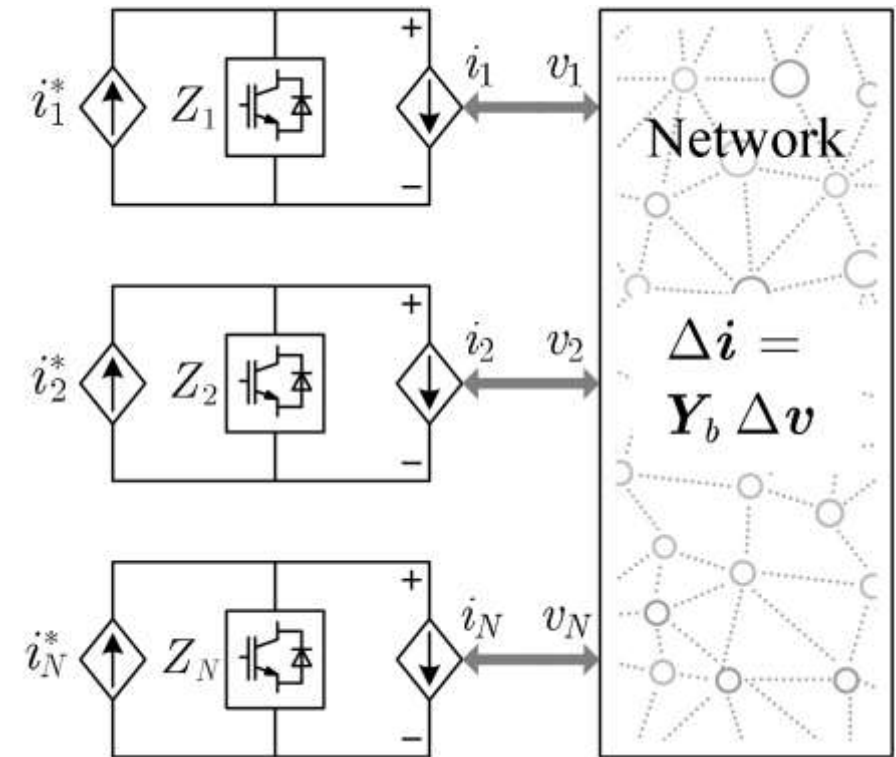


# Two Physical Views

## Mechanical-Centric: Torque Coefficient



## Electronic-Centric: Impedance/Admittance





# Two Mathematical Tools

## State-Space: Preserve Structure

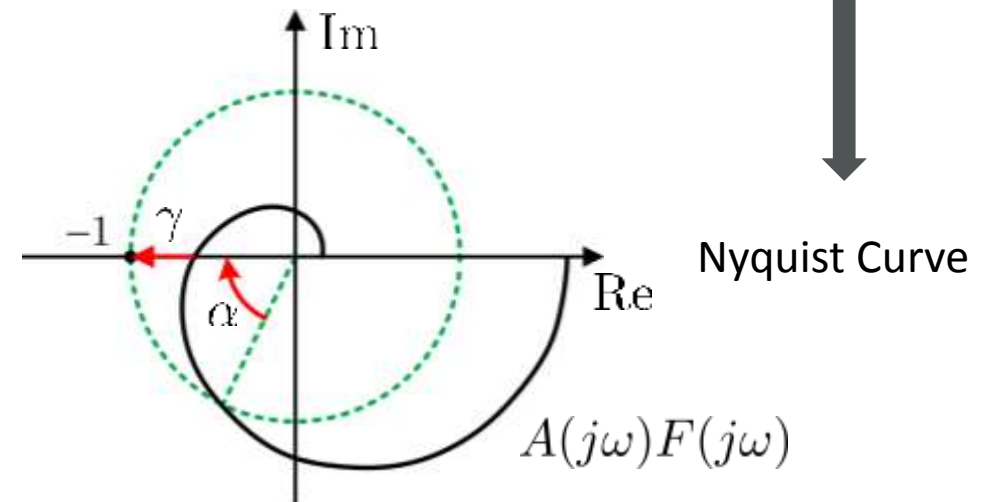
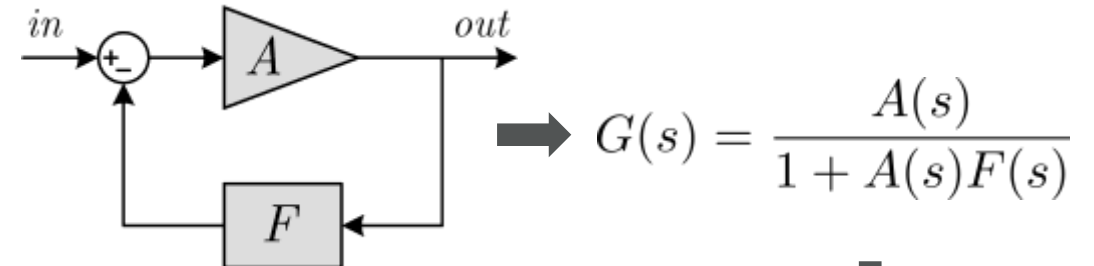
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_3 \end{pmatrix}$$

$$\updownarrow \quad x = \Phi z, \quad z = \Psi x$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} \lambda_1 \text{ Eigenvalue} & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_3 \end{pmatrix}$$

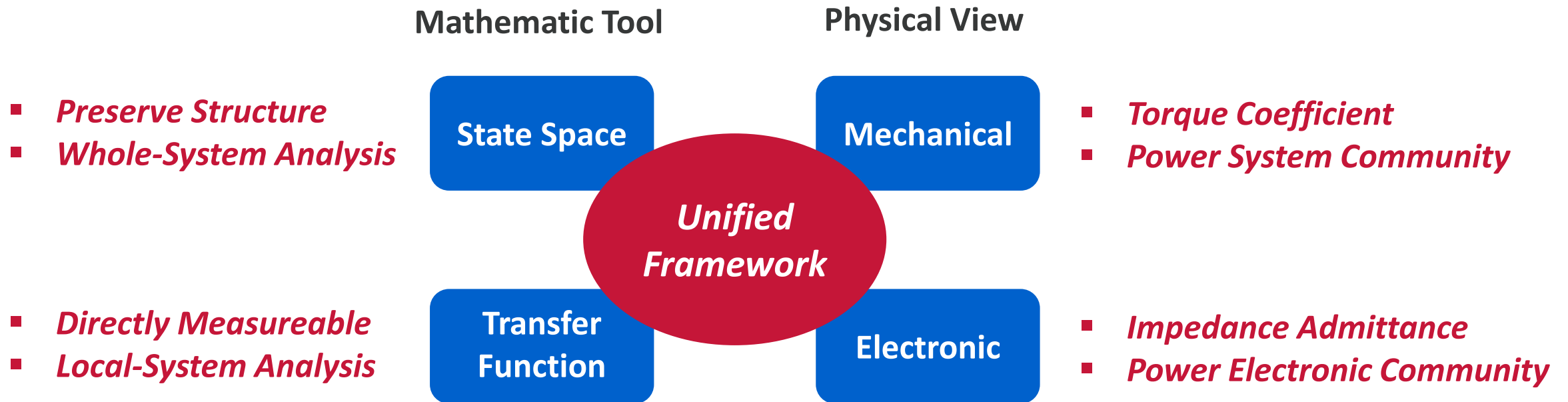
$$p_{nk} = \phi_{nk} \psi_{nk} \quad \textbf{Participation Factor}$$

## Transfer-Function: Directly Measurable

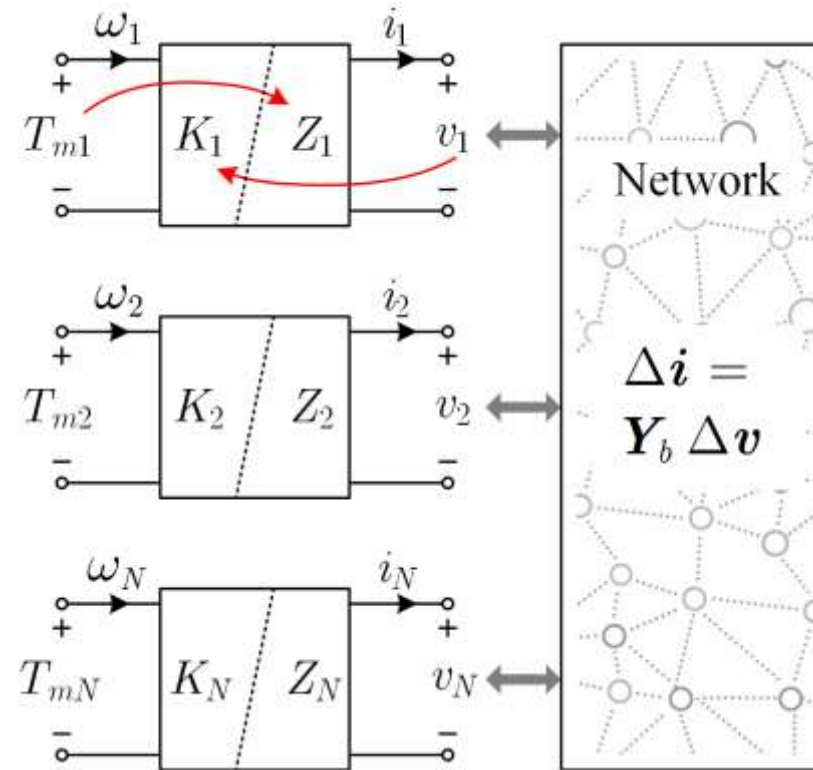
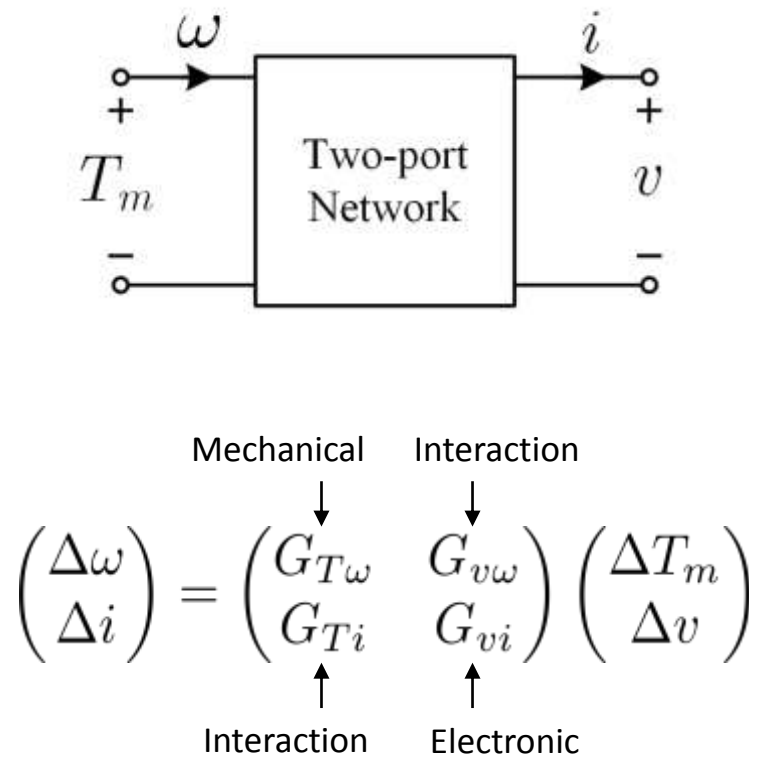


$$\alpha, \gamma \quad \textbf{Phase Margin, Gain Margin}$$

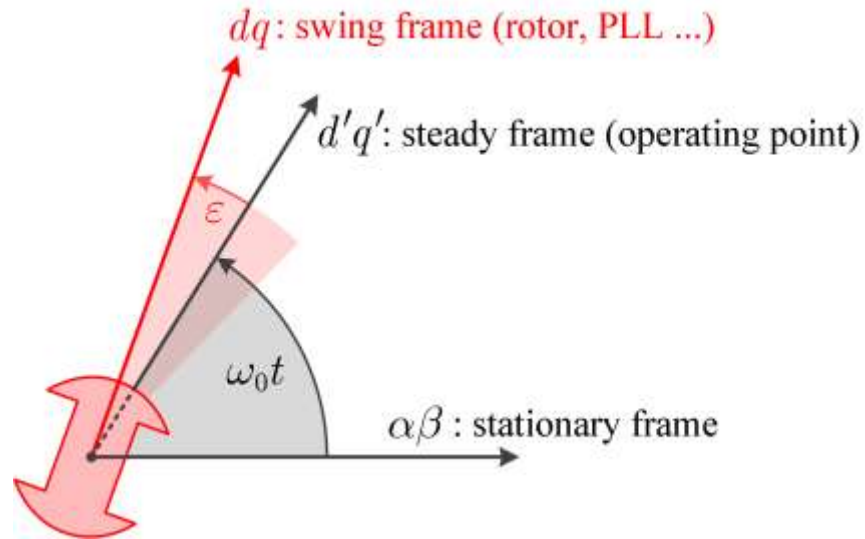
# Unified Framework for Whole System Integration



# Unification of Mechanical and Electronic Views



# Mechanical → Electronic



Reflecting mechanical dynamics in impedance/admittance

$$\begin{pmatrix} \Delta\omega \\ \Delta i \end{pmatrix} = \begin{pmatrix} G_{T\omega} & G_{v\omega} \\ G_{Ti} & G_{vi} \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta v \end{pmatrix}$$



$$\begin{pmatrix} \Delta\omega \\ \Delta i' - I_0 \frac{\Delta\omega}{s} \end{pmatrix} = \begin{pmatrix} G_{T\omega} & G_{v\omega} \\ G_{Ti} & G_{vi} \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta v' - V_0 \frac{\Delta\omega}{s} \end{pmatrix}$$



$$\begin{pmatrix} \Delta\omega \\ \Delta i' \end{pmatrix} = \begin{pmatrix} G'_{T\omega} & G'_{v\omega} \\ G'_{Ti} & G'_{vi} \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta v' \end{pmatrix}$$



$$G'_{vi} = G_{vi} + (I_0 - G_{vi} V_0)(s + G_{v\omega} V_0)^{-1} G_{v\omega}$$



$$Z' = Y'^{-1} = -G'^{-1}_{vi}$$

# Electronic → Mechanical

Reflecting electronic dynamics in torque coefficient

$$\begin{pmatrix} \Delta\omega \\ \Delta i' \end{pmatrix} = \begin{pmatrix} G'_{T\omega} & G'_{v\omega} \\ G'_{Ti} & G'_{vi} \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta v' \end{pmatrix}$$

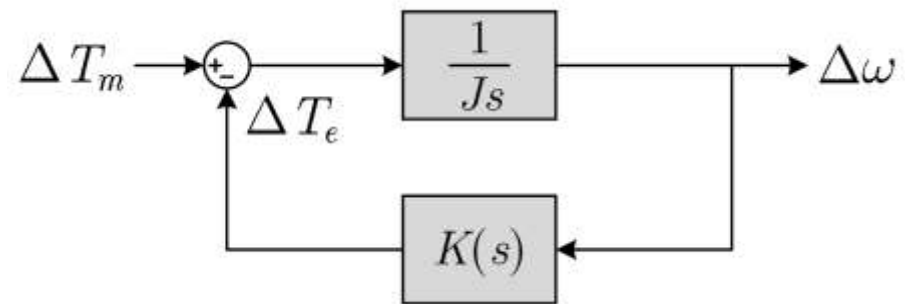
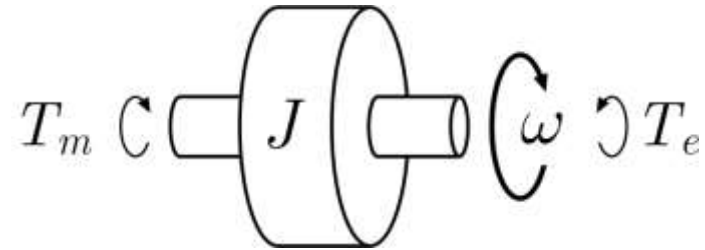
$\Delta i' = Y'_{sys} \Delta v'$



$$G''_{T\omega} = G'_{T\omega} + G'_{v\omega} (\mathbf{Y}'_{sys} + \mathbf{Y}')^{-1} G'_{Ti}$$



$$K(s) = G''_{T\omega}{}^{-1} - Js$$



# Unification of State-Space and Transfer-Function Methods

$$\dot{x} = Ax$$



$$\dot{x} = Ax + Bu$$

$$y = Cx$$



$$x = \Phi z$$

$$\dot{z} = \bar{A}z + \bar{B}u$$

$$y = \bar{C}z$$

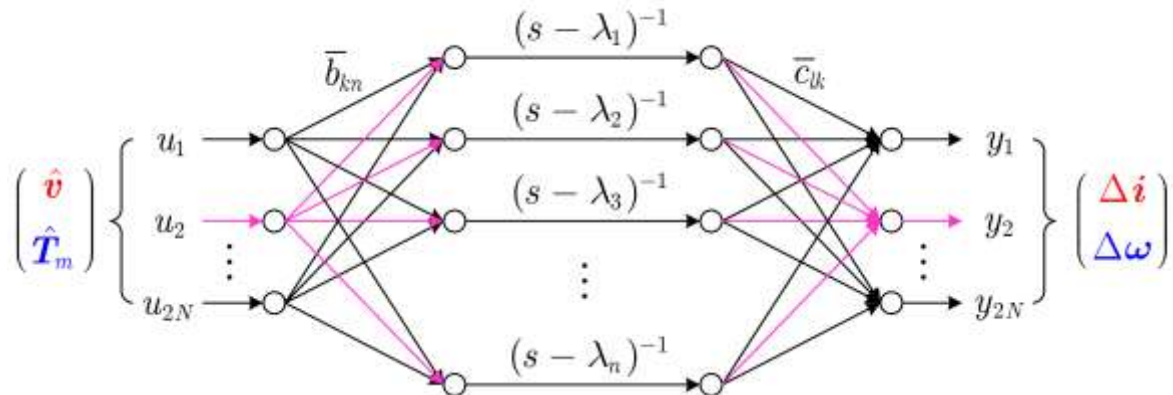
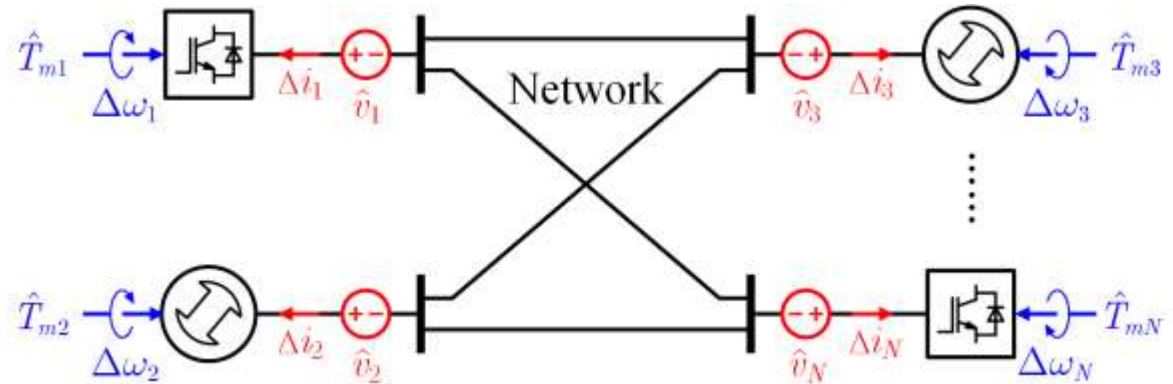


$$G_{u_n \rightarrow y_l} = \sum_k \frac{\bar{c}_{lk} \bar{b}_{kn}}{s - \lambda_k}$$

$$u = \begin{pmatrix} \hat{v} \\ \hat{T}_m \end{pmatrix} \quad y = \begin{pmatrix} \Delta i \\ \Delta \omega \end{pmatrix}$$



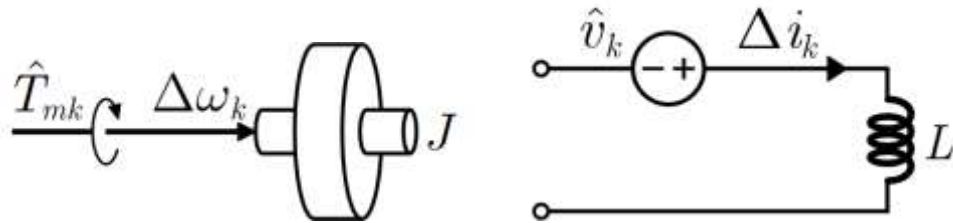
Residue





# Residue is Participation Factor

Input and output are acting on the same apparent state



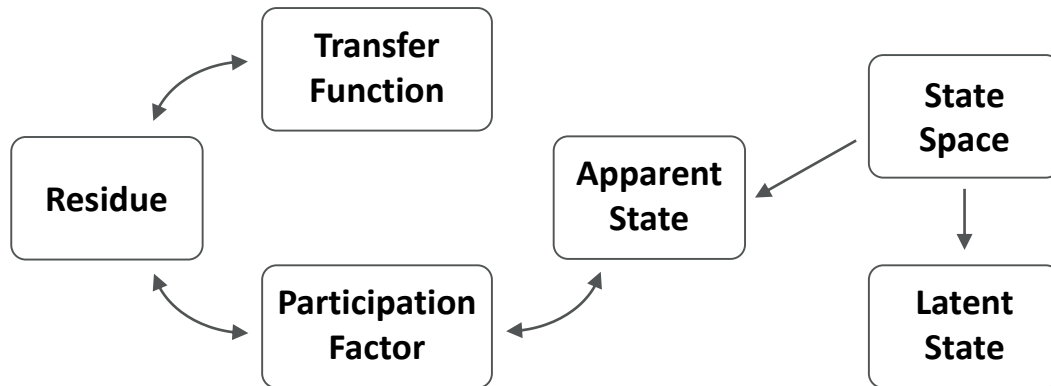
Apparent State  
↓

$$\begin{cases} B_{:n} = (0, \dots, 0, b_n, 0, \dots, 0)^\top \\ C_n = (0, \dots, 0, c_n, 0, \dots, 0) \end{cases}$$

$$\bar{c}_{nk} = c_n \phi_{rk}, \bar{b}_{kn} = b_n \psi_{kr}$$

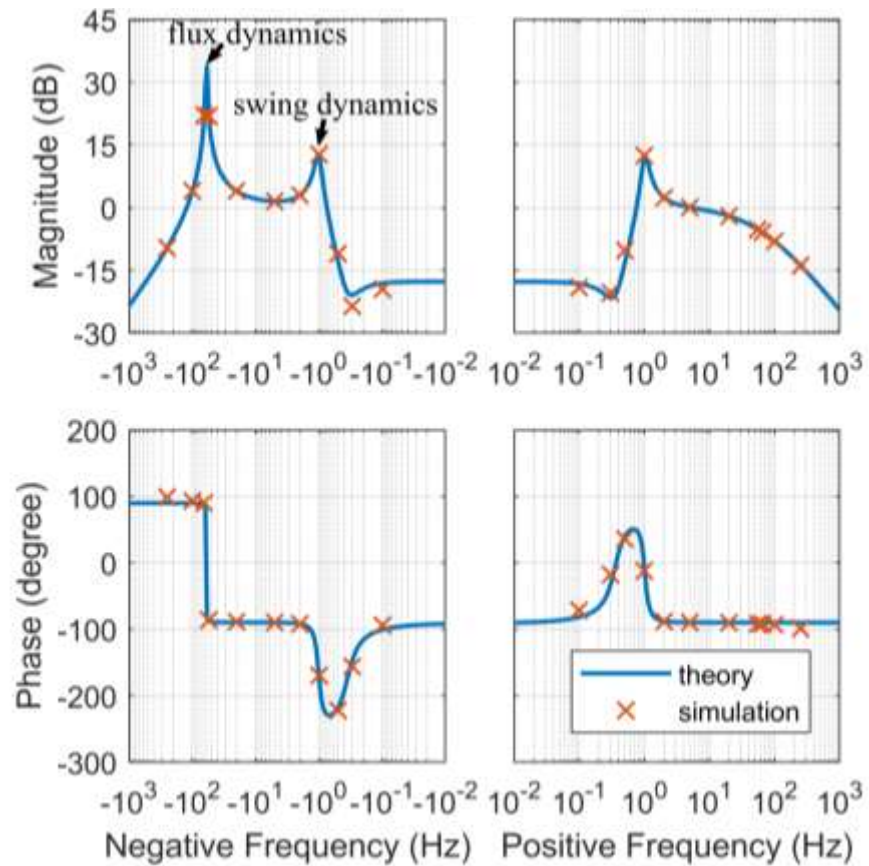
$$G_{u_n \rightarrow y_n} = \sum_k \frac{\bar{c}_{nk} \bar{b}_{kn}}{s - \lambda_k} = c_n b_n \sum_k \frac{p_{rk}}{s - \lambda_k}$$

Residue is equivalent to the participation factor !

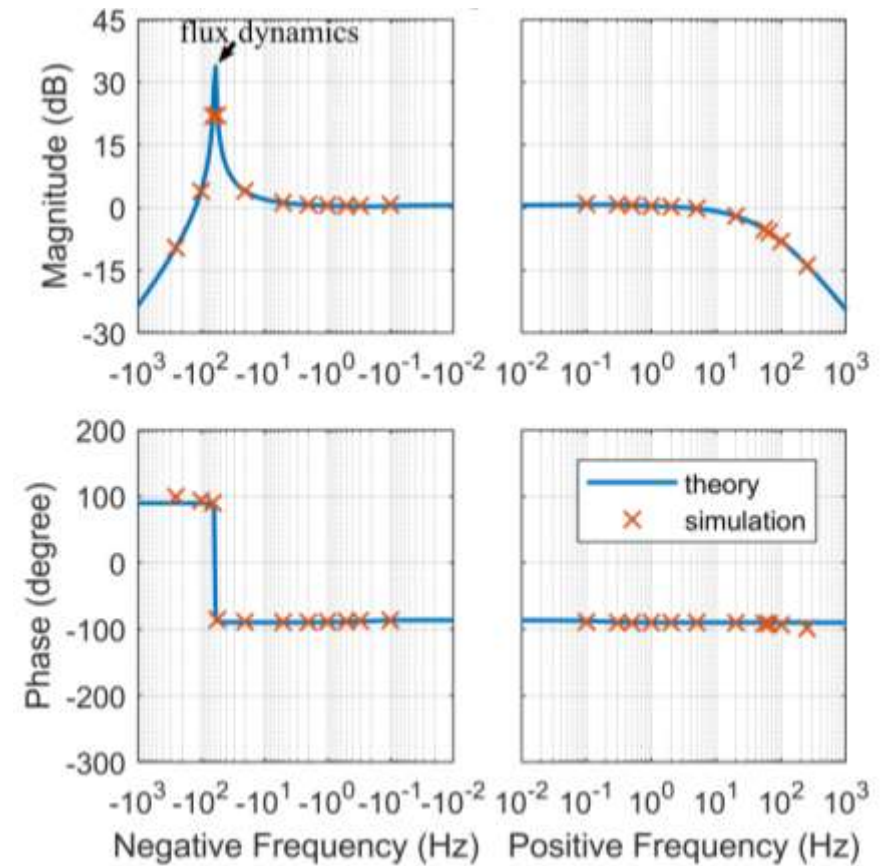


# Example: Synchronous Generator

$Y'_{sg}$  : mechanical dynamics embedded

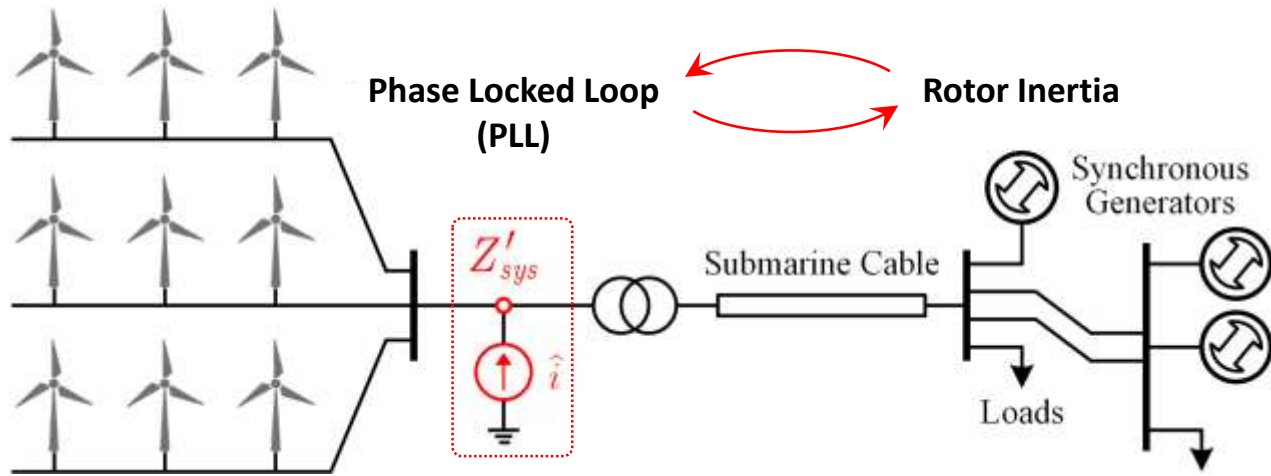


$Y_{sg}$  : mechanical dynamics not embedded

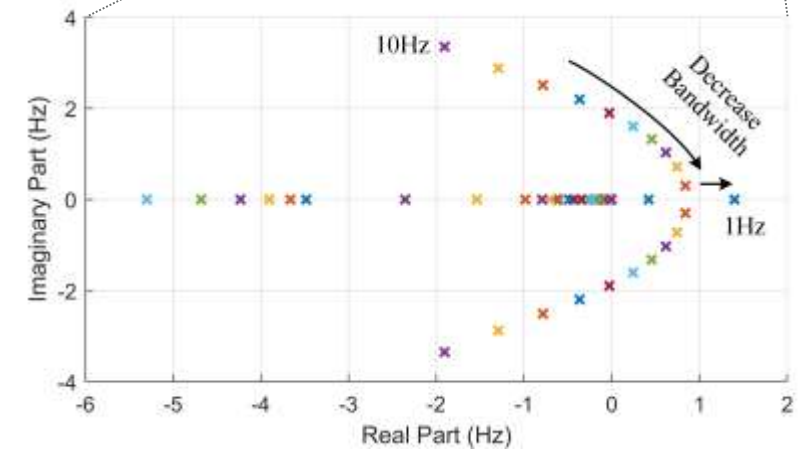
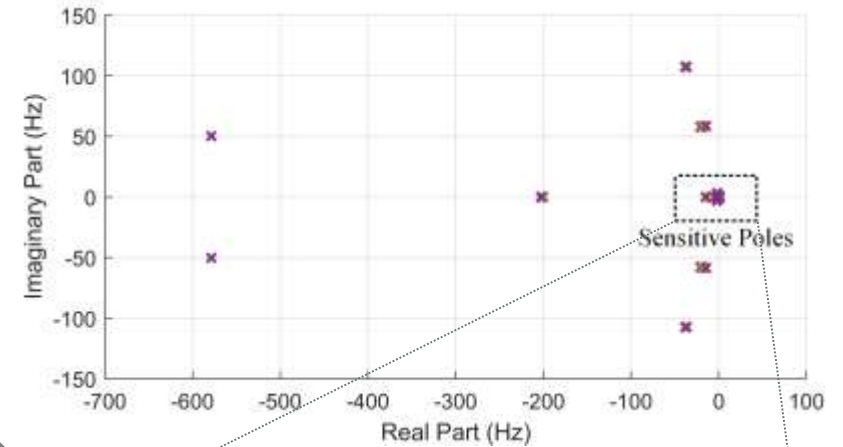


Admittance of a synchronous generator with and without mechanical dynamics reflected

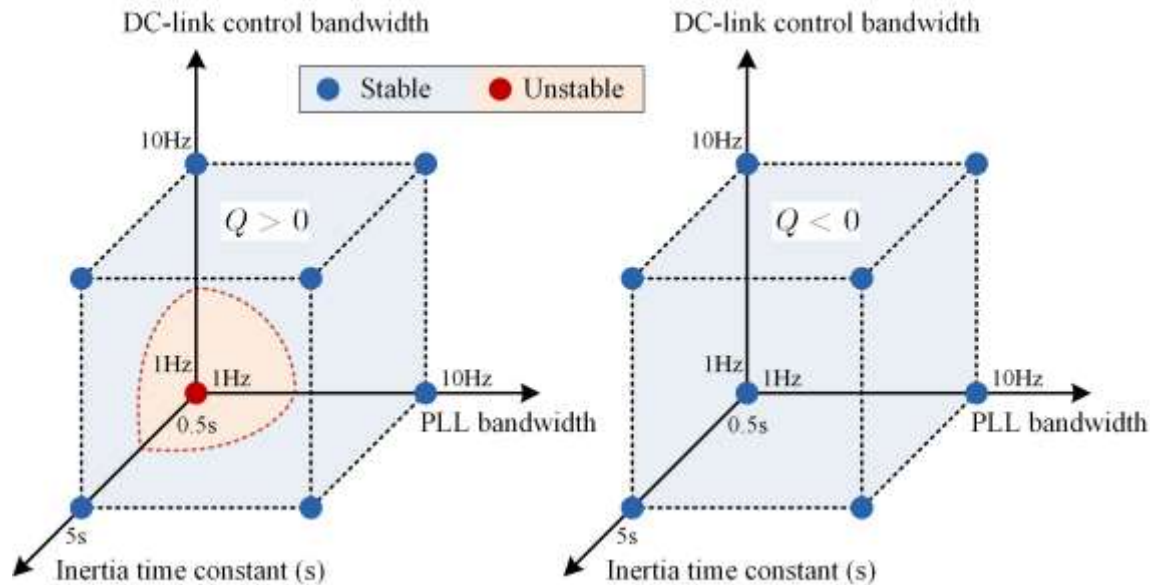
# PLL-Inertia Interaction : Z-Representation (Electronic)



Root Locus of Whole-System Impedance  $Z'_{sys}$



# PLL-Inertia Interaction : Z-Representation (Electronic)



Impedance of a synchronous generator with mechanical dynamics

$$Z'_{sg} = \overbrace{\begin{pmatrix} Z_L(s+1) & \\ & Z_L(s-1) \end{pmatrix}}^{\text{electronic}} + \overbrace{\frac{1}{M(s)} \begin{pmatrix} s+1 & s+1 \\ s-1 & s-1 \end{pmatrix}}^{\text{mechanical}}$$



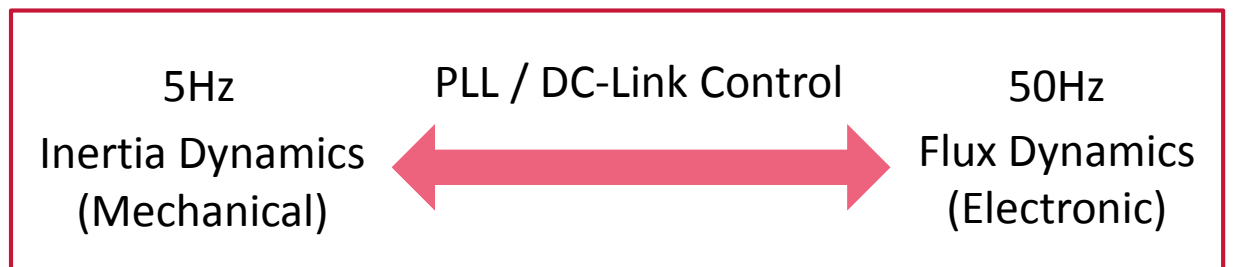
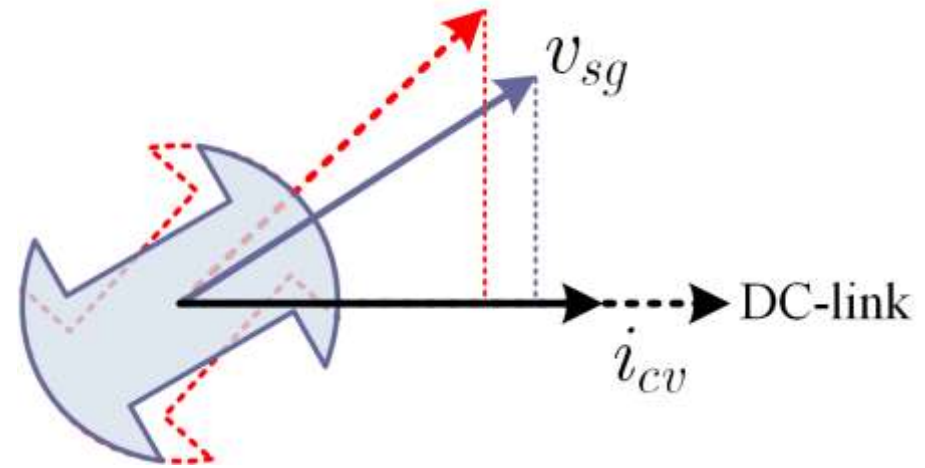
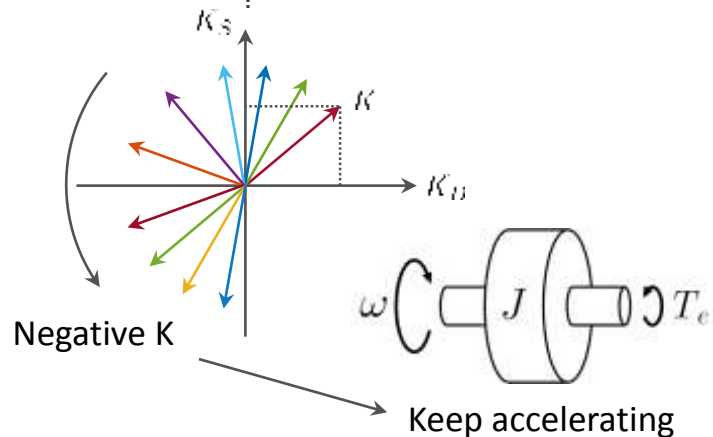
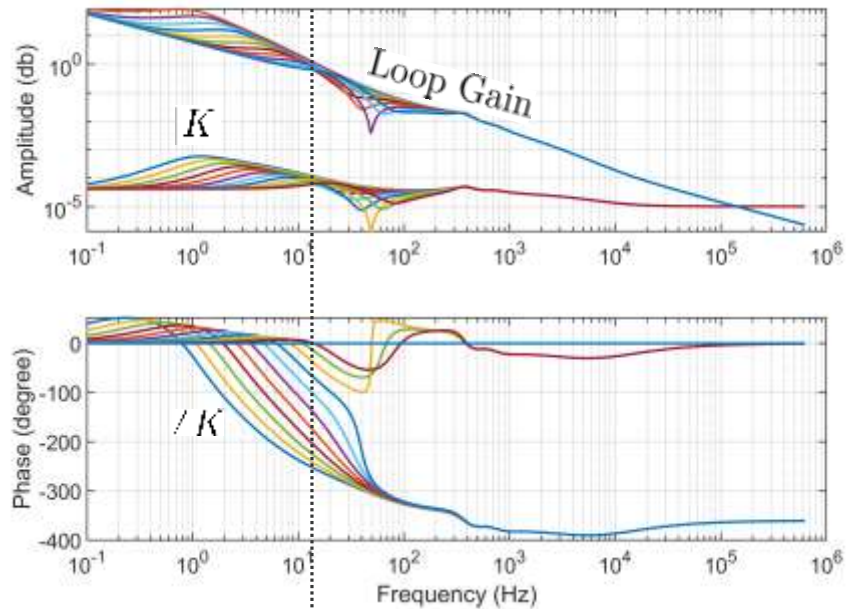
$$M(s) = 2/\psi_f^2 \cdot (Js^2 + Ds - i_{q0}\psi_f)$$

Poles in the right half plane

- Synchronous generator impedance has unstable poles at positive reactive power (over excitation)
- Converter  $\approx$  current source when PLL and dc-link control is slower than inertia timescale

# PLL-Inertia Interaction : K-Representation (Mechanical)

Bode Plot of Torque Coefficient K



**Thank You!**

